

The Finite-time Ruin Probability in the Presence of Heavy-tailed Claims and Dependent Return Rates on Risky Investment

Qihe Tang¹ and Raluca Vernic²

¹ Department of Statistics and Actuarial Science, The University of Iowa

E-mail: qtang@stat.uiowa.edu

² Faculty of Mathematics and Computer Science, Ovidius University of Constanta

E-mail: rvernic@univ-ovidius.ro

March 15, 2010

Abstract

Consider a non-standard discrete-time insurance risk model in which periodic claim amounts and premium incomes form a sequence of i.i.d. random vectors, but the claim amount and premium income in a given period are dependent. The insurer is allowed to invest a constant fraction of his/her wealth in a risky stock and keep the remaining wealth in a risk-free bond. Assume that the claim sizes follow a heavy-tailed distribution and that the return rates of the stock are dependent according to a suitably chosen multivariate distribution. We focus on the calculation of the finite-time ruin probability. As main results, we derive some exact asymptotic formulas which capture the impacts of the underlying dependence structures. As an application, we determine an asymptotic value of the fraction invested to the risky stock to maximize the expected terminal wealth of the insurer under the constraint that the ruin probability is bounded by a small constant.

Keywords: Asymptotics; Optimal investment strategy; Pareto-like distributions; Tail probability.

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